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Evgenii Mikhailovich Semenov (on his 60th birthday)

In August 2000, Evgenii Mikhailovich Semenov, doctor of the physical and mathematical sciences and professor of Voronezh State University, became sixty years old.

He was born on 22 August 1940 in Grozny. His father, Mikhael Kuz'mich Semenov (1913–1941), was the acting army commander at a frontier post on the Romanian border and was killed in the war at the age of 27. After moving to Voronezh, his mother, Nina Kapitonovna Vitchenko (1906–1976), worked as an assistant in Voronezh State University.

The whole life of Semenov has been connected with Voronezh State University. The Voronezh mathematical school was flourishing during his student years (1957–62), and his teachers included such well-known mathematicians as Yu. G. Borisovich, M. A. Krasnosel'skii, S. G. Krein, A. E. Medvedev, B. S. Mityagin, V. M. Tikhomirov, V. I. Sobolev, and A. S. Shvarts. His mathematical gifts manifested themselves while he was still a student. Under the influence of Krein and Krasnosel'skii he became interested in functional analysis and obtained his first results. This was at a time (end of 1950s, beginning of 1960s) when a new part of functional analysis, the theory of interpolation operators, was developing rapidly. Due principally to the work of Krein, Voronezh became one of the main centres for this promising area. The abstract theory of interpolation was established at the end of the 1950s and the beginning of the 1960s, principally by J.-L. Lions, A. P. Calderón, and S. G. Krein, and important contributions to its development were made by N. A. Aronszajn, E. Gagliardo and J. Peetre.

The theory of scales of Banach spaces and their interpolation properties were studied in papers of Krein and Petunin. It is no wonder then that the first papers [1], [2] of Semenov, who was Krein's student, were devoted to this topic. On the basis of this work he wrote his candidate's dissertation "Scales of Banach spaces connecting the spaces L_1 and L_∞", which he successfully defended in 1964.

At the same time Semenov started to pay more attention to Banach spaces of measurable functions in their own right and not just in connection with interpolation theory. In [3] he introduced the important idea of a symmetric space.

This is a Banach-ideal function space with the following property: if $y(t) \in E$ and the functions $|y(t)|$ and $|x(t)|$ are equi-measurable, that is,

$$\operatorname{meas}\{t : |y(t)| > z\} = \operatorname{meas}\{t : |x(t)| > z\} \qquad (z > 0),$$

then $x(t) \in E$ and $\|x\|_E = \|y\|_E$.

Important examples of symmetric spaces are the L_p-spaces for $1 \leqslant p \leqslant \infty$, their generalization the Orlicz spaces and also the Lorentz and Marcinkiewicz spaces.

The idea of a symmetric space turned out later to be very fruitful, both for the study of interpolation properties of operators and for the study of the geometry of function spaces. It transpired that many problems achieve their most complete and even definitive solution precisely for this class of spaces.

In 1968, at the age of 28, Semenov successfully defended his doctoral dissertation "Interpolation of linear operators in symmetric spaces". In 1967 he was one of the first in the country to be awarded a prize of the Lenin Youth Organization.

We shall mention here one of the most interesting results of this period. It concerns a famous theorem of Marcinkiewicz:

Let the quasi-linear operator A map L_{p_i} into the space $S(V, d\nu)$ of measurable almost everywhere finite functions on some measure space and suppose that

$$z\,\nu\{v \in V : |Ax(v)| > z\}^{1/q_i} \leqslant M_i \|x\|_{L_{p_i}} \qquad (i = 0, 1)$$

for all $x = x(t) \in L_{p_i}$ and $z > 0$. If $1 \leqslant p_i \leqslant q_i < \infty$ $(i = 0, 1)$, then A maps L_p into L_q, where

$$p^{-1} = (1-\theta)p_0^{-1} + \theta p_1^{-1} \quad \text{and} \quad q^{-1} = (1-\theta)q_0^{-1} + \theta q_1^{-1} \quad (0 < \theta < 1).$$

This theorem stimulated a large number of investigations concerned with generalizations and refinements. One of the most interesting and complete results was obtained in joint work with S. G. Krein [9] (a first version for the case of exponential functions having been obtained by Semenov in [6]). We shall state it.

Let $\kappa(t)$, $\delta(t)$ be non-negative measurable functions with $\delta([0,1]) \subset [0,1]$. If E is a symmetric space on $[0,1]$, then we denote by $E_{\kappa,\delta}$ the set of all functions $x = x(t)$ for which

$$\|x\|_{\kappa,\delta} = \|x^{**}(\delta(t))\kappa(t)\|_E < \infty,$$

where

$$x^{**}(s) = \frac{1}{s}\int_0^s x^*(u)\,du,$$

and $x^*(u)$ is the non-increasing rearrangement of the function $|x(u)|$.

Let φ_0, φ_1, ψ_0, ψ_1 be non-negative concave functions such that the set of values of $\psi_0(s)/\psi_1(s)$ contains the set of values of $\varphi_0(s)/\varphi_1(s)$ and

$$\lim_{t\to 0+} \frac{\ln \sup_s(\varphi_1(ts)/\varphi_1(s))}{\ln t} < \alpha_E \leqslant \beta_E < \lim_{t\to\infty} \frac{\ln \sup_s(\varphi_0(ts)/\varphi_0(s))}{\ln t}\,,$$

where α_E and β_E are the Boyd indices of the symmetric space E.

If A is a continuous linear operator from the Lorentz space $\Lambda(\varphi_i)$ into the Marcinkiewicz space $M(\psi_i)$, then A is also a continuous operator from E to $E_{\kappa,\delta}$, where $\delta(t)$ is a measurable solution of the equation.

$$\frac{\psi_0(\delta(t))}{\psi_1(\delta(t))} = \frac{\varphi_0(t)}{\varphi_1(t)},$$

and $\kappa(t) = \psi_0(\delta(t))/\varphi_0(t)$. We note that under quite general assumptions the space $E_{\kappa,\delta}$ constructed is optimal (that is, it cannot be made smaller).

The first monograph [15] in the Soviet literature on interpolation of operators was published in 1978. Its authors—S. G. Krein, Yu. I. Petunin and E. M. Semenov—gave an account of certain developments in this part of functional analysis, and set the path for future work. This book differed from other monographs on interpolation theory in both the choice of topics and the way they were covered; it plays a continuing important role in the development of this theory. It has been translated into English and published in the USA.

One of the volumes in the series "Achievements of Science and Technology", is a survey, published in 1986, of the theory of interpolation of operators [26]. Semenov was one of the authors (together with Yu. I. Brudnyi and S. G. Krein).

Semenov's papers on the geometry of function spaces form an important part of his research work. His interests in this area are very broad. He seeks definitive and precise results (necessary and sufficient conditions, full descriptions of spaces with given properties, and so on).

In his joint papers with V. A. Rodin [10], [16] he obtained important extremal properties of the space G which is the closure of the space L_∞ in the Orlicz space L_M^* with $M(u) = \exp(u^2) - 1$.

A significant development in the geometry of Banach spaces in the second half of the twentieth century was concerned with the notions of Rademacher type and cotype, introduced in 1972 by Hoffmann-Jørgenson and Maurey.

A Banach space is said to have cotype q (respectively, the Orlicz q-property) if there exists a positive constant C such that for any elements $x_1, x_2, \dots, x_n \in X$ one has

$$\biggl(\sum_{i=1}^n \|x_i\|^q\biggr)^{1/q} \leqslant C \int_0^1 \biggl\|\sum_{i=1}^n r_i(t)x_i\biggr\|\,dt$$

$$\biggl(\text{respectively,}\qquad \biggl(\sum_{i=1}^n \|x_i\|^q\biggr)^{1/q} \leqslant C \max_{\varepsilon_i=\pm1}\biggl\|\sum_{i=1}^n \varepsilon_i x_i\biggr\|\biggr).$$

For a long time it was unknown whether the classes of Banach spaces with cotype 2 and the Orlicz 2-property coincide. In [29] it was proved that these spaces do coincide, for all symmetric function spaces.

Semenov successfully applied the methods of interpolation theory to solve problems in the theory of orthogonal series.

In [10] he studied properties of series in the Rademacher system belonging to various classes of symmetric spaces. Extending the set of function spaces led to the discovery of a previously unknown phenomenon in which a function space is uniquely determined by the requirement of membership in it of the sum of a

Rademacher series. Moreover, conditions on the sum of the series were obtained that are necessary and sufficient for the coefficients to belong to l_p, for $1 < p < 2$. This work stimulated a whole series of investigations of a similar type.

The recent papers of Semenov and his students confirm the fruitfulness of his ideas and methods.

A special place in his research is occupied by his work on the Haar system. His results are described in the monograph [37] written jointly with I. Ya. Novikov. The main feature is the broad use of operator theory methods: the properties of the Haar system are studied in terms of properties of operators naturally connected with this system.

Semenov is the author of over 130 papers and two monographs. His results have gained wide international recognition and they are frequently cited by Russian and foreign mathematicians. He collaborates actively and fruitfully with mathematicians from various countries.

He took an active part in the organization of the well-known Voronezh winter mathematical schools. Mathematicians from many Russian universities have attended his lectures. His fluent English makes for easy communication with foreign mathematicians. He has taken part in many international conferences and given talks at universities in Italy, Spain and America.

Semenov is a kind and modest person, ready at any time to talk about mathematics, and this inevitably attracts to him both mature mathematicians and students and research students. Mountaineering forms a part of his healthy life-style, and this too appeals to the young mathematicians whose education he influences. About twenty of his students were candidates and three of them (S. V. Astashkin, M. Sh. Braverman, V. A. Rodin) became doctors of the physical and mathematical sciences.

Since 1971 he has been a professor and head of the group on the theory of functions and geometry at Voronezh University, from 1978 to 1983 he was head of the Department of Mathematics, and since 1974 he has been the research head of the operator equations section of the Mathematics Research Institute at Voronezh University.

Evgenii Mikhailovich Semenov is celebrating his sixtieth birthday at the peak of his creative powers, and he has many new ideas and projects. We wish him good health, future mathematical successes, talented students, and the best of luck for the realization of all his plans.

S. V. Astashkin, S. M. Nikol'skii, S. Ya. Novikov

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